



Legged State Estimation

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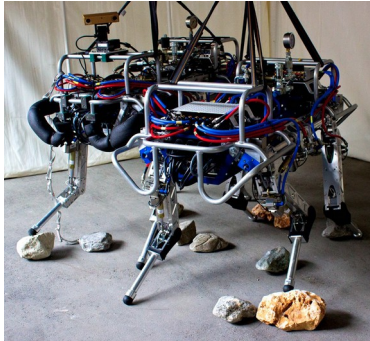
Thursday 31st, 2019

- Intro Proprioceptive State Estimation
- Inertial-Legged Estimation with Pronto
- Pronto Applications
- Other methods

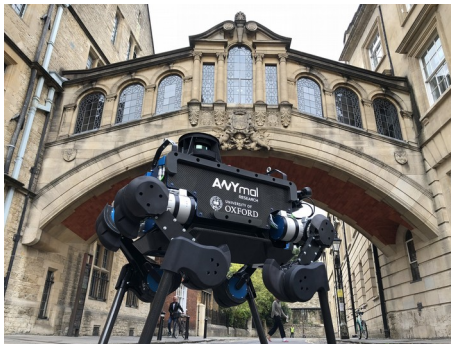
About me



2014 – 2018: PhD/Postdoc at IIT (Genoa, Italy)



2018: moved to ORI for Postdoc

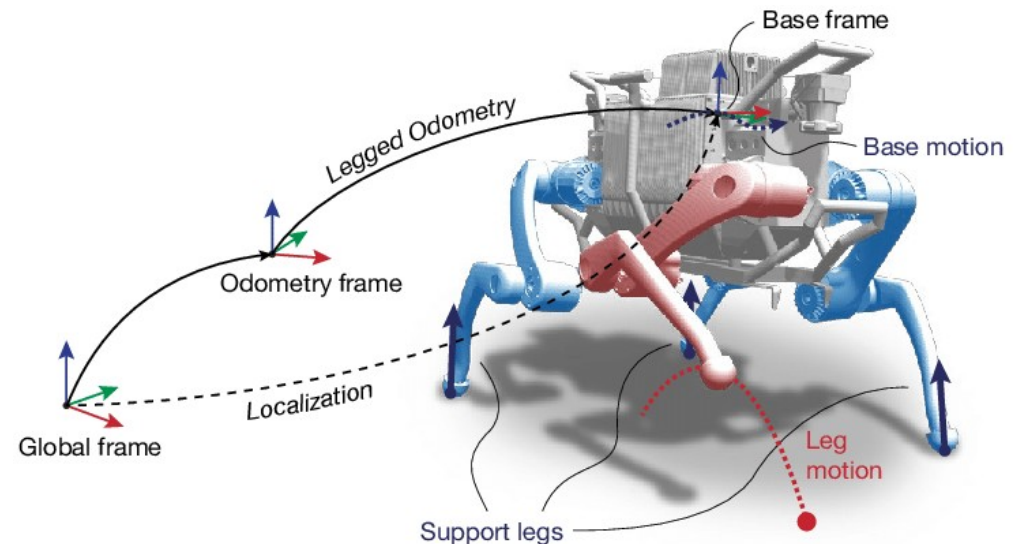


- **Intro Proprioceptive State Estimation**
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What is state estimation?

Estimate the **position** and **velocity** of a floating base legged robot from sensor measurements

- High frequency ($\sim 0.2-1$ kHz)
→ **control**
- Low frequency ($\sim 1-90$ Hz)
→ **localization, mapping, path planning**



Proprio- vs. Extero-ceptive

Proprioceptive state estimation → properties of the robot



NOTE:
images
not in
scale

Exteroceptive state estimation → properties of the environment



NOTE:
images
not in
scale

Normally, **both are used** either in a **loosely or tightly coupled** system

Strapdown = IMU on the chassis

$$\Theta_k = \Theta_{k-1} \exp(\hat{\omega}_k \Delta t)$$

$$\Theta_k = \Theta_{k-1} \left(I_3 + \frac{\sin \sigma}{\sigma} \hat{\omega}_k + \frac{1 - \cos \sigma}{\sigma^2} (\hat{\omega}_k)^2 \right)$$

$$\ddot{\mathbf{x}}_k = \Theta_k \mathbf{a}_k - w \mathbf{g}$$

$$\dot{\mathbf{x}}_k = \dot{\mathbf{x}}_{k-1} + \Delta t \ddot{\mathbf{x}}_k$$

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \Delta t \dot{\mathbf{x}}_k$$

$$\hat{\omega}^\wedge(t) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & \omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$



What is Leg Odometry?



Estimation of incremental motion of a legged robot through kinematics (and dynamics) assuming no slip

- **How to detect contacts**
 - Measured force (force/torque sensors)
 - Estimated force (from dynamics)
 - other
- **How to fuse measurements**
 - No fusion
 - EKF, UKF, xKF
 - Optimization (QP, TSIF, F. Graph)
- **Meas. model/State Definition**
 - Foot positions in state
 - Velocity updates
- **Leg fusion**
 - Multiple legs
 - One leg

What is Leg Odometry?



- Quasi-Static motion
- Six legs \rightarrow overconstr.
- One Swing leg
- No fusion with IMU

$$\min \sum_j w_j \mathbf{e}_j^T \mathbf{e}_j$$

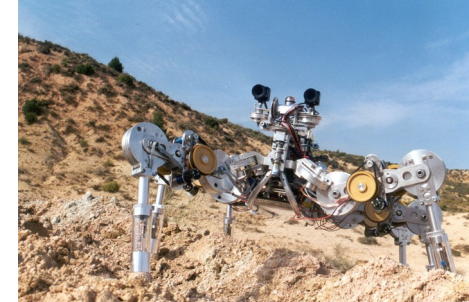
$$\mathbf{e}_j = {}^W \mathbf{p}_j - {}^W R_B {}^B \mathbf{p}_j + {}^W \mathbf{t}$$

Early State Estimation (90s/10s)

- Six legs (tripod gait)
- Flat Ground
- Mild GPS/VO integr.
- MoCap aid



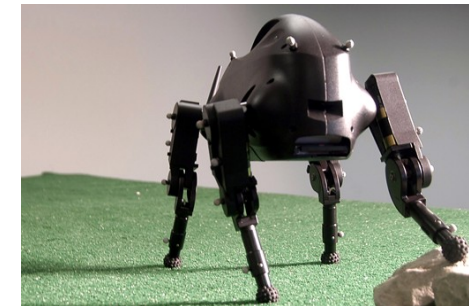
Crawler - DLR



Lauron III - FIK



RHex - BDI

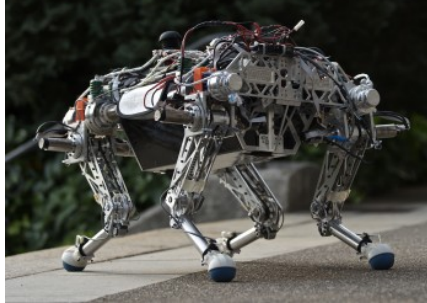


LittleDog - BDI

Modern State Estimation ('10-now)



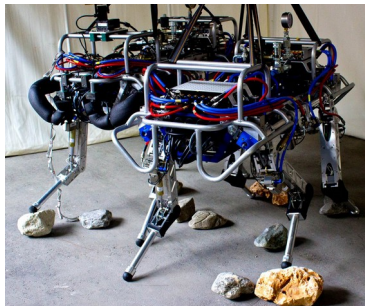
LS3 - BDI



StarLETH - ETHZ



BigDog - BDI



HyQ - IIT

- Inertial-Kinematics
- Kalman Filter-based
- No assumptions on leg configuration or gait
- (w/o) contact sensors
- Exteroceptive fusion

State Estimation on Humanoids

- Additional joint (ankle)
- Flat foot (foot orientation, torque)
- 4 force/torque sensors at the foot/spring at ankle
- Non-negligible flexibilities



- **Extended/Unscented Kalman Filter** → Inertial Process Model + Leg Odometry
- **Dynamic State Estimation via Quadratic Programming** → minimize model/process error
- **Two State Implicit Filter** → no prediction phase

- Absolute position and yaw are unobservable
- Roll and pitch are observable → gravity
- Twist is measured/observable → IMU

Position and yaw drift without exteroceptive updates

We can limit the drift rate ...

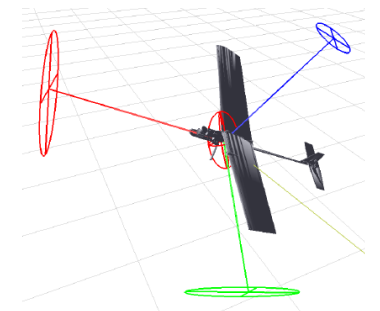
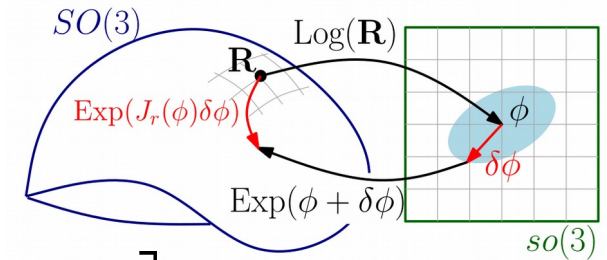
“State Estimation for Legged Robots – Consistent Fusion of Leg Kinematics and IMU”

M. Bloesch, M. Hutter, M.A. Hoepflinger, S. Leutenegger, C. Gehring, C.D. Remy, R. Siegwart – RSS 2012

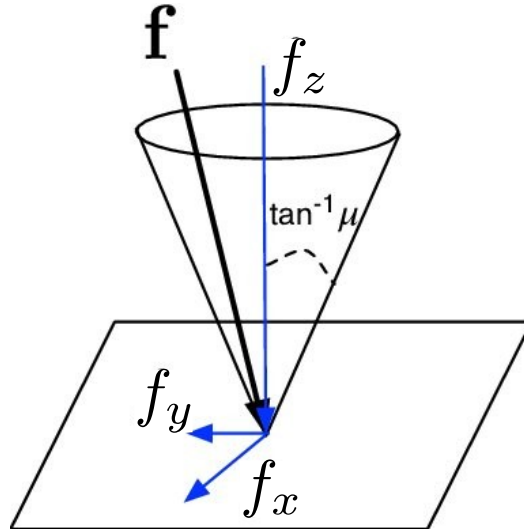
- How to parametrize orientation?
 - Rotation Matrix / Quaternion for state, Euler for error

$$\mathbf{u}_k = \begin{bmatrix} \tilde{\boldsymbol{\omega}}_k \\ \tilde{\ddot{\mathbf{x}}}_k \end{bmatrix} = \begin{bmatrix} R_i^b({}_i\boldsymbol{\omega}_i - \mathbf{b}_{\omega,k-1}^+) \\ R_i^b({}_i\ddot{\mathbf{x}}_i - \mathbf{b}_{a,k-1}^+) \end{bmatrix}$$

$$\mathbf{x}_k^- = \begin{bmatrix} \mathbf{x}_k^- \\ \dot{\mathbf{x}}_k^- \\ \Theta_k^- \\ a \mathbf{b}_k^- \\ \omega \mathbf{b}_k^- \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{k-1}^+ \\ \dot{\mathbf{x}}_{k-1}^+ \\ \mathbf{0} \\ a \mathbf{b}_{k-1}^+ \\ \omega \mathbf{b}_{k-1}^+ \end{bmatrix} + \begin{bmatrix} \dot{\mathbf{x}}_{k-1}^+ \Delta t \\ (-\tilde{\boldsymbol{\omega}}_k \times \dot{\mathbf{x}}_{k-1}^+ + (\Theta_{k-1}^+)^{-1} {}_w\mathbf{g} + \tilde{\ddot{\mathbf{x}}}_k) \Delta t \\ \Theta_{k-1}^+ \exp(\boldsymbol{\omega}_k^\wedge \Delta t) \\ \mathbf{b}_{a,k-1}^- \Delta t \\ \dot{\mathbf{b}}_{\omega,k-1}^- \Delta t \end{bmatrix}$$



Contact Assumptions



No slippage

$$\frac{\sqrt{f_x^2 + f_y^2}}{f_z} \leq \mu$$

No rotation (flat foot)

$$\begin{bmatrix} -\tau_y / f_z \\ \tau_x / f_z \end{bmatrix} \leq \begin{bmatrix} c_x \\ c_y \end{bmatrix}$$
$$|\tau_z| \leq \mu_z f_z$$

No contact sensors

$$\mathbf{f} = -(J_{cq}^T(\mathbf{q}))^\dagger (\boldsymbol{\tau} - \mathbf{h}_q - F^T_b \ddot{\mathbf{x}}_b)$$

Leg Odometry (velocity)



- Base velocity from one leg

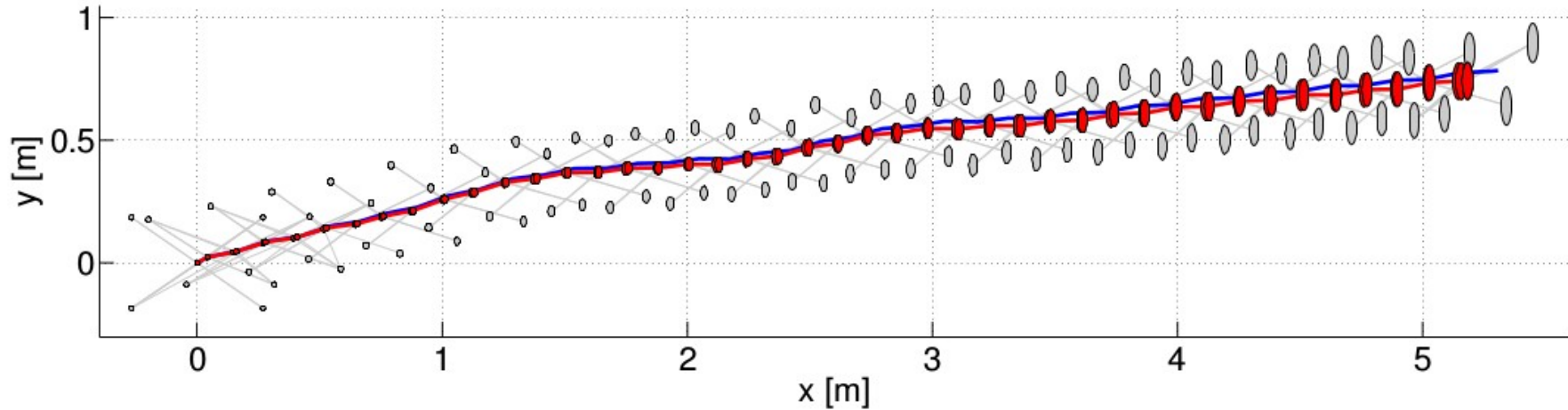
$$\mathbf{x} = [\mathbf{x}, \dot{\mathbf{x}}, \boldsymbol{\theta}, \mathbf{b}_{\omega}, \mathbf{b}_a]$$

$${}^b\dot{\mathbf{x}}_b = -{}^b\dot{\mathbf{x}}_f - {}^b\boldsymbol{\omega}_b \times {}^b\mathbf{x}_f$$

Leg Odometry (position)

Include the contact points in the state:

$$\mathbf{x} = [\mathbf{x}, \dot{\mathbf{x}}, \boldsymbol{\theta}, \mathbf{p}_1, \dots, \mathbf{p}_4, \mathbf{b}_\omega, \mathbf{b}_a]$$

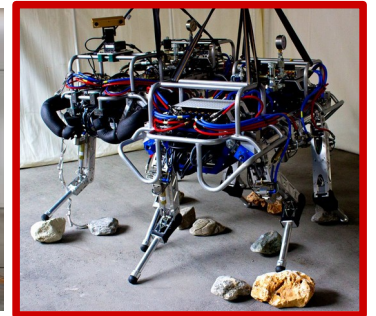
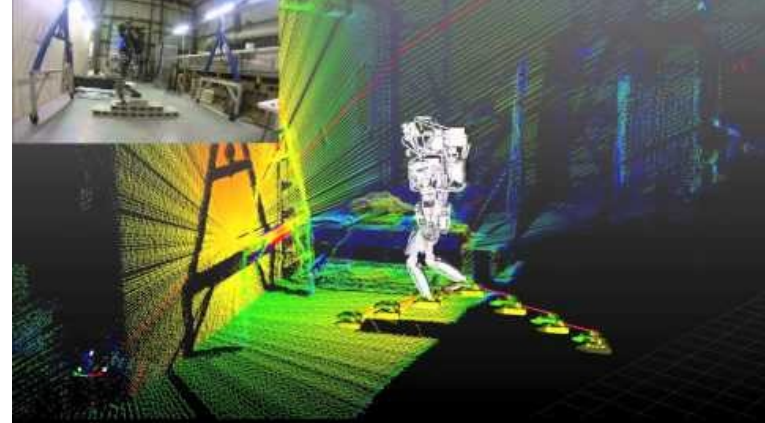


“State Estimation for Legged Robots – Consistent Fusion of Leg Kinematics and IMU”

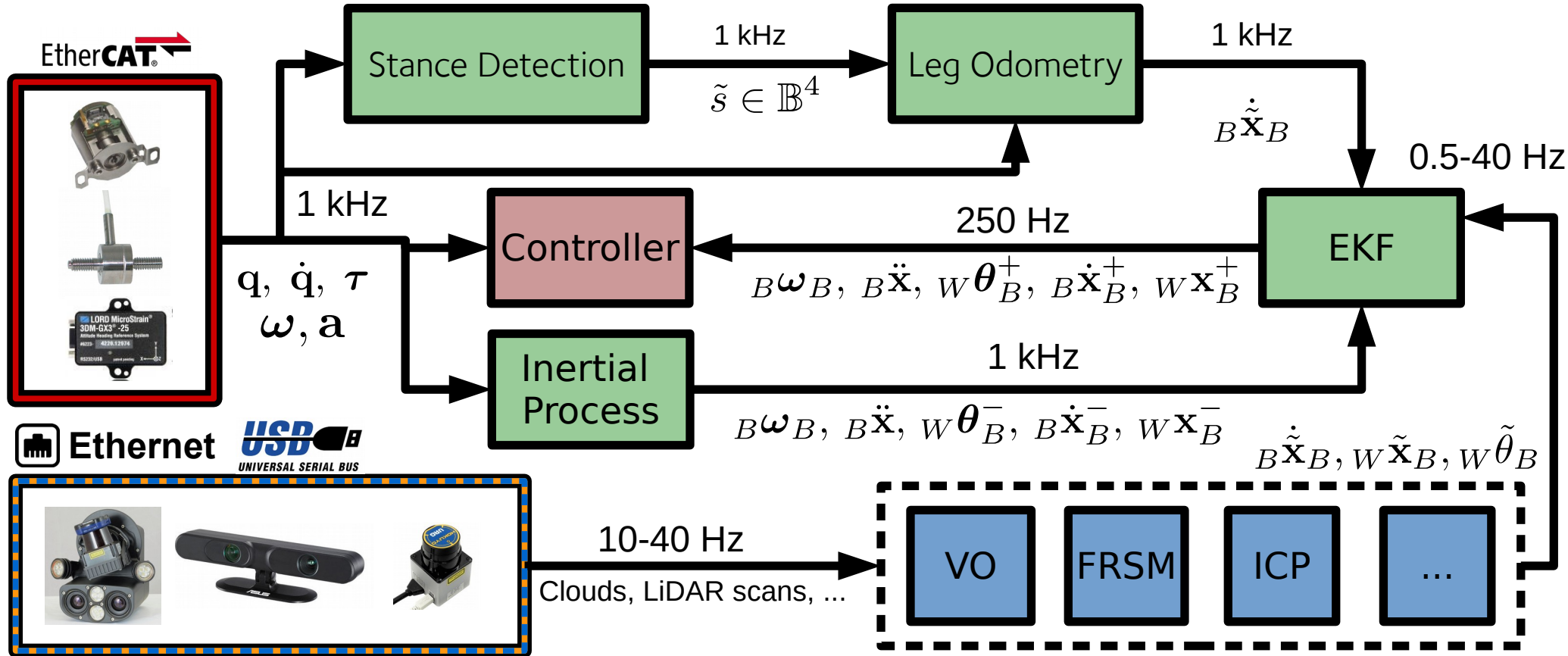
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- Extended Kalman Filter
- History of Measurements
- Modular
- Multi-platform

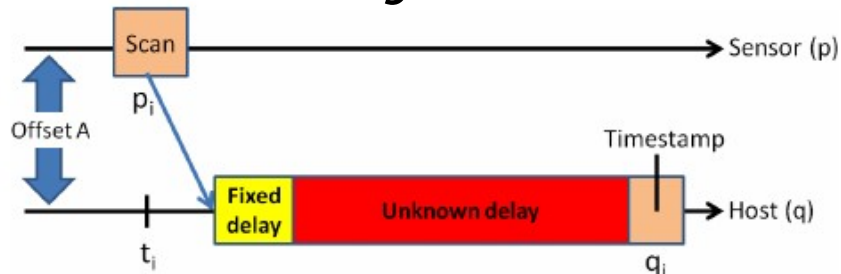


State Estimation Scheme

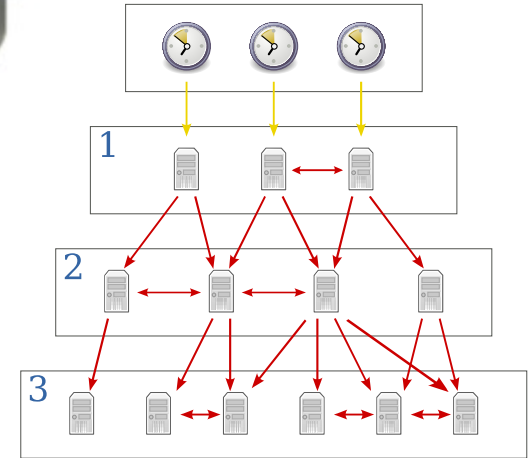


Time Synchronization

- EtherCAT
- FPGA
- NTP
- Passive Synchronization



A passive solution to the sensor synchronization problem
E. Olson – IROS 2010



Challenges in Leg Odometry

- Kinematics (joint position, joint velocity)
- IMU (angular velocity, acceleration)

$${}^b\dot{\mathbf{X}}_b = -{}^b\dot{\mathbf{X}}_f - {}^b\boldsymbol{\omega}_b \times {}^b\mathbf{X}_f$$

1. Probability of Reliable Contact
2. Measurement Update (from multiple legs)
3. Covariance estimation (for the EKF)

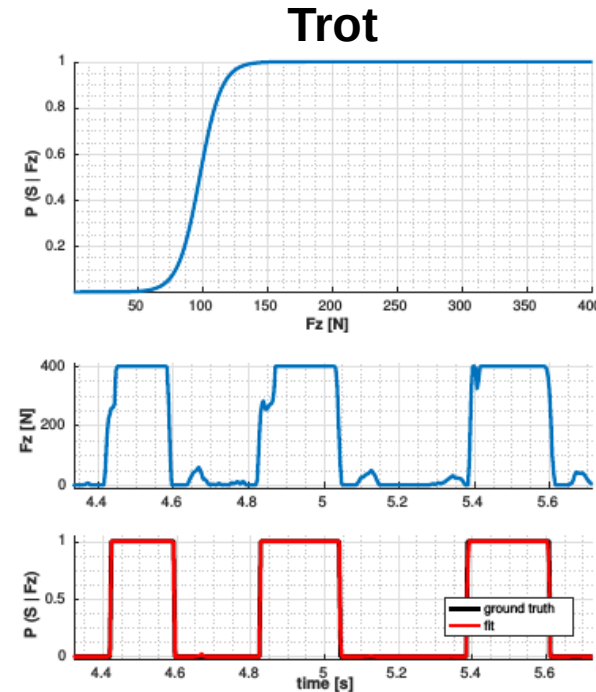
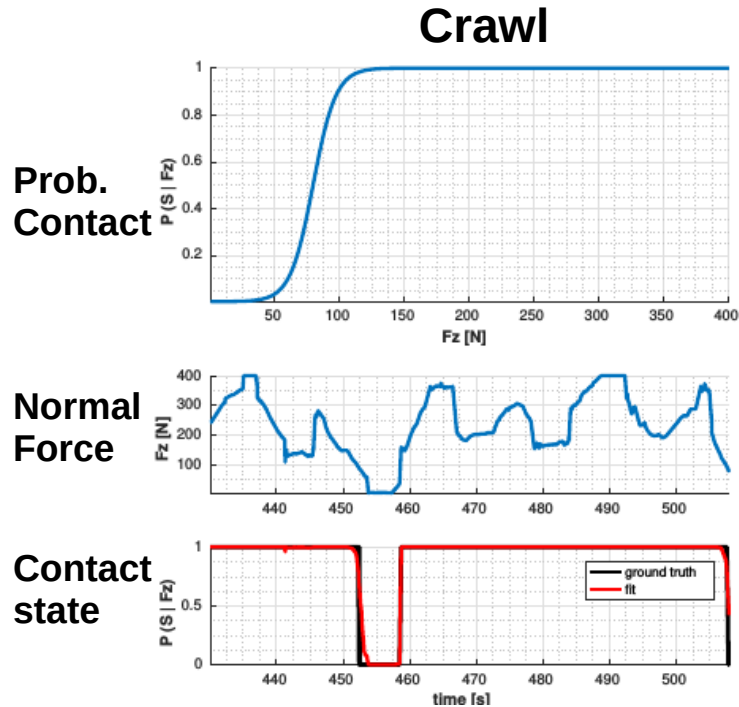


Probabilistic Contact Estimation and Impact Detection for State Estimation of Quadruped Robots

M. Camurri, M. Fallon, S. Bazeille, A. Radulescu, V. Barasuol, D.G. Caldwell, C. Semini

IEEE Robotics and Automation Letters (RA-L), 2017

Which legs are in reliable contact?



- No foot sensors
→ estimate GRF from joint efforts
- What threshold?
→ minimize velocity error

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Probability of contact as weighting criteria:

$$\dot{\mathbf{x}}_k = \frac{\sum_{i \in C} \mathcal{P}_k(s_l = 1 | \mathbf{f}_l^k) \dot{\mathbf{x}}_{k,l}}{\sum_{i \in C} \mathcal{P}_k(s_l = 1 | \mathbf{f}_l^k)}$$

$$\mathcal{P}_k(s_l = 1 | \mathbf{f}_l^k) = \frac{1}{1 + \exp(-\beta f_{z,l}^k - \beta_0)}$$

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How to calculate covariance?



Inter-Leg covariance

$$\Lambda(\sigma_{e,x}, \sigma_{e,y}, \sigma_{e,z})^2 \approx \frac{1}{\dim(C)} \sum_{l \in C} (\dot{\mathbf{x}}_{k,l} - \dot{\mathbf{x}}_k)(\dot{\mathbf{x}}_{k,l} - \dot{\mathbf{x}}_k)^T$$

Impact

$$\Delta_{k,f} = |\Delta \bar{f}_z^k| = \frac{1}{\dim(C)} \sum_{l \in C} |f_{z,l}^k - f_{z,l}^{k-1}|$$

Final estimate

$$\Sigma_v = \Sigma_0 + \left[\frac{1}{2} \Lambda(\sigma_{e,x}, \sigma_{e,y}, \sigma_{e,z}) + I_3 \frac{\Delta_{k,f}}{2\alpha} \right]^2$$

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How to calculate covariance?

$$\Sigma_v = \Sigma_0 + \underbrace{\left[\frac{1}{2} \Lambda(\sigma_{e,x}, \sigma_{e,y}, \sigma_{e,z}) \right]}_{\text{Leg agreement}} + \underbrace{\left[I_3 \frac{\Delta_{k,f}}{2\alpha} \right]}_{\text{Impact}} \Bigg]^2$$

$|\Delta \bar{f}_z^k| = \frac{1}{\dim(C)} \sum_{i \in C} |f_{z,l}^k - f_{z,l}^{k-1}|$

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IEEE Robotics and Automation Letters, 2017

How to calculate covariance?

Covariance boost

Include force discontinuity in the covariance

Dark blue

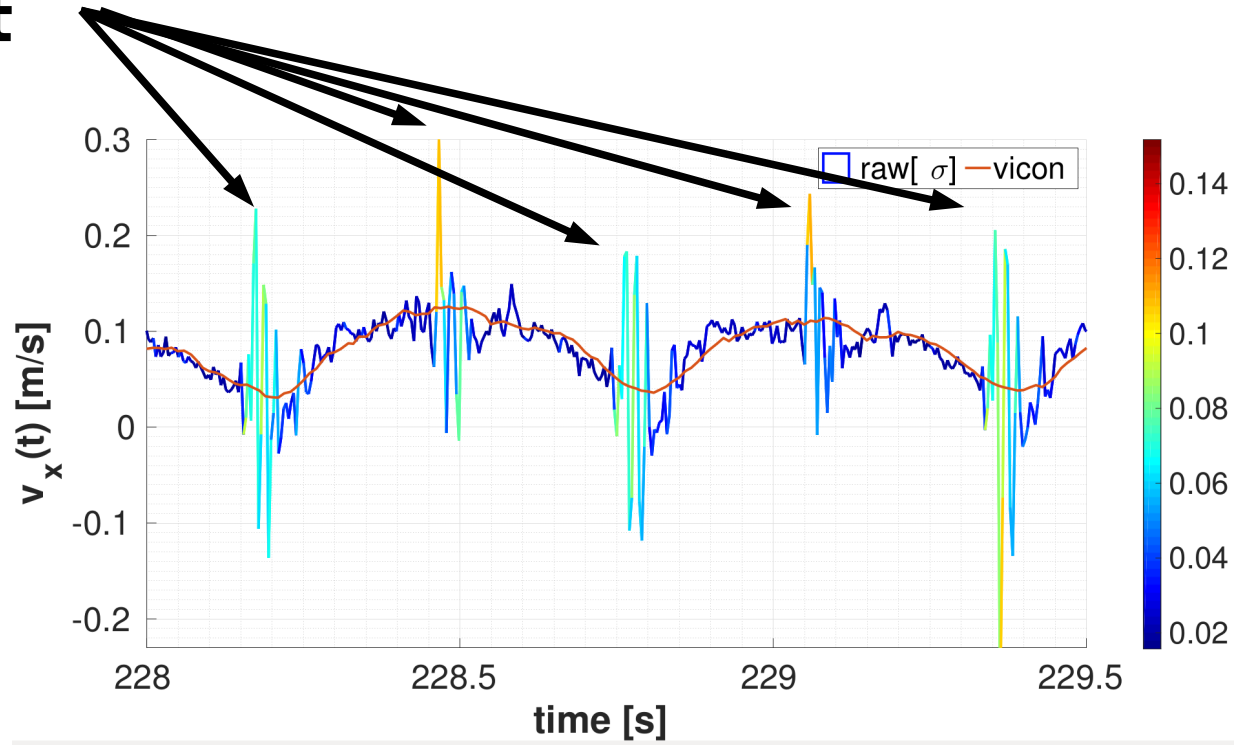
=

low uncertainty

Azure/orange

=

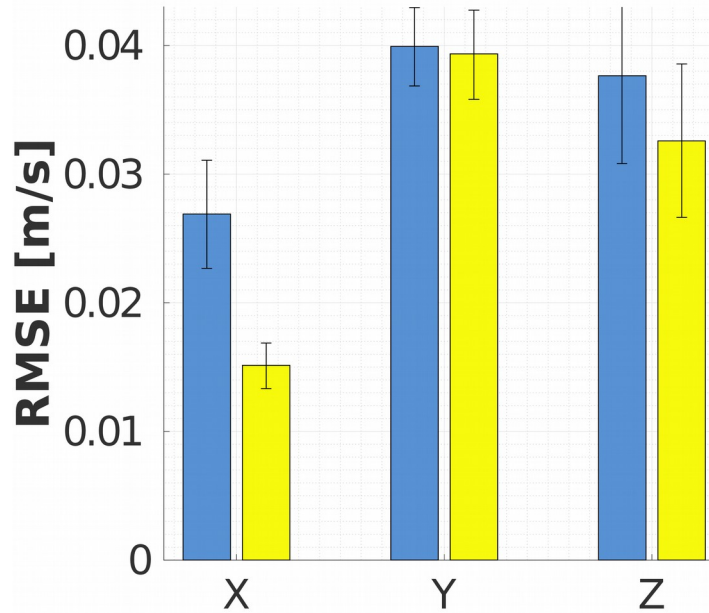
high uncertainty



Experimental Results: Velocity

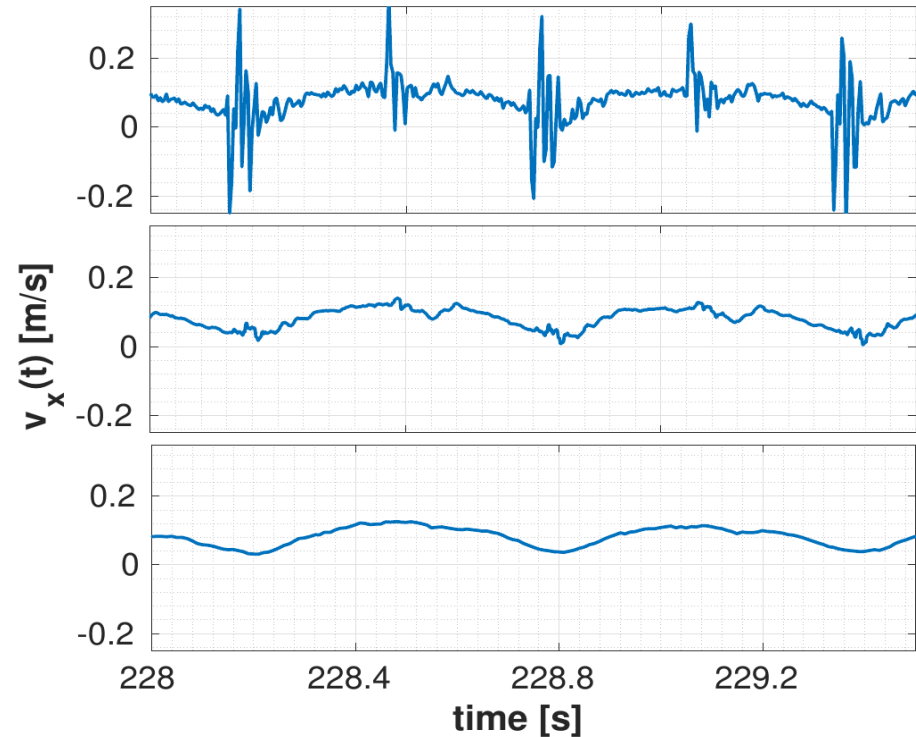
Fixed threshold vs. our approach

RMSE Trot

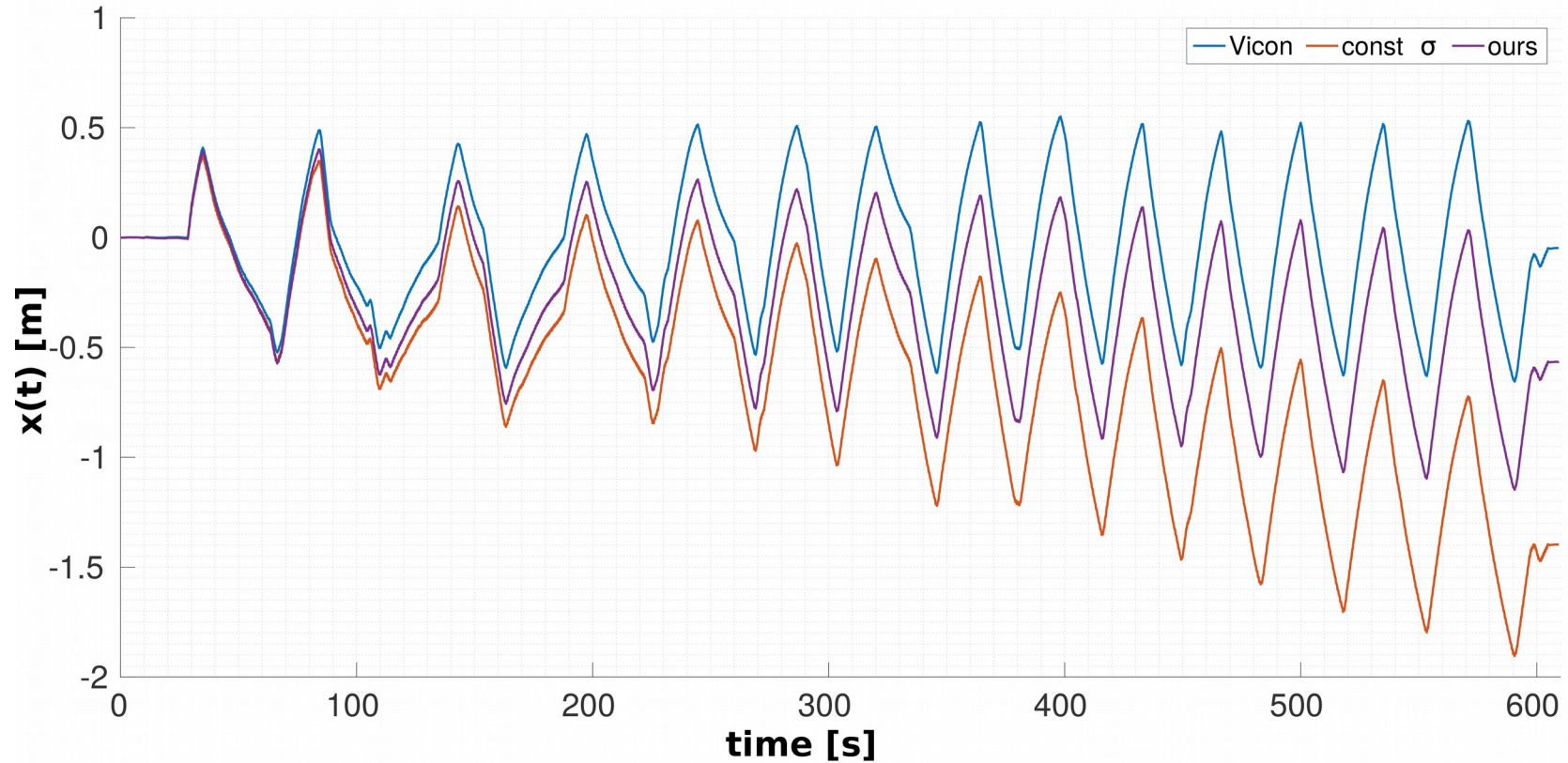


>1h logs, multiple gaits

Velocity forward direction raw vs. filtered vs. ground truth

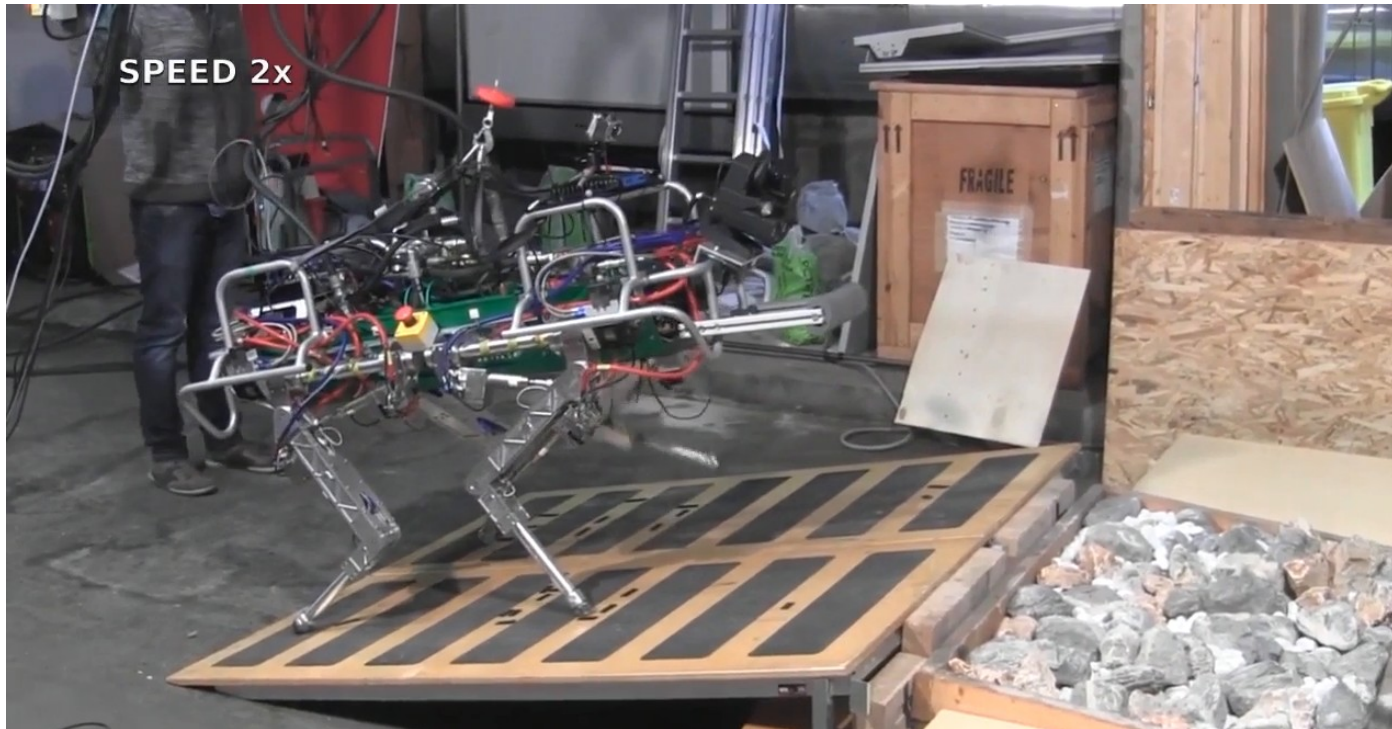


Experimental Results: Position



- Intro Proprioceptive State Estimation
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Statically Stable Crawl with height/step reflex

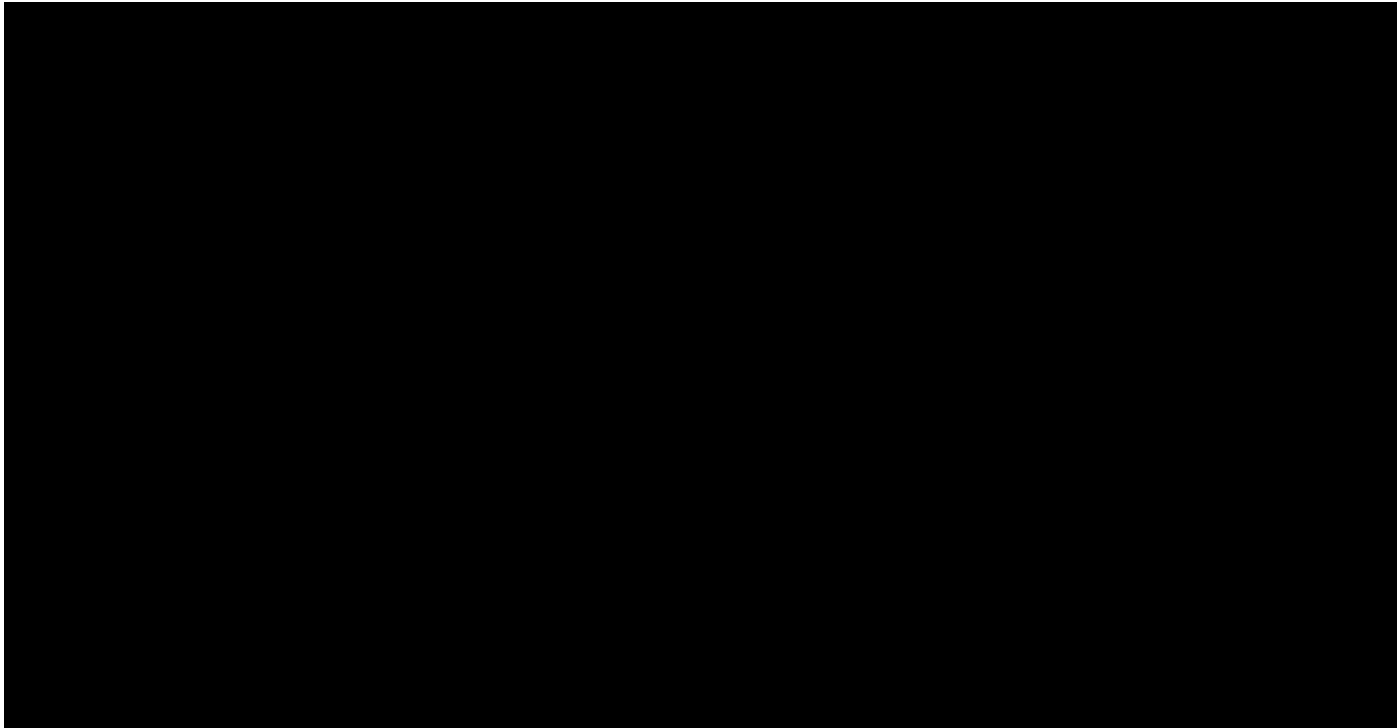


“Heuristic Planning for Rough Terrain Locomotion in Presence of External Disturbances and Variable Perception Quality”

M. Focchi, R. Orsolino, M. Camurri, V. Barasuol, C. Mastalli, D.G. Caldwell, C. Semini

To appear in “Springer Tracts in Advanced Robotics”
preprint:
arxiv.org/abs/1805.10238

Reactive Trotting + CNNs on local height maps



**“Fast and Continuous
Foothold Adaptation for
Dynamic Locomotion
through Convolutional
Neural Networks”**

O. Villarreal, V. Barasuol, M. Camurri, M. Focchi, L. Franceschi, M. Pontil, D.G. Caldwell, C. Semini

Accepted to RA-L / ICRA 19,
preprint:
arxiv.org/abs/1809.09759

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- **Extended/Unscented Kalman Filter** → Inertial Process Model + Leg Odometry
- **Dynamic State Estimation via Quadratic Programming** → minimize model/process error
- **Two State Implicit Filter** → no prediction phase

QP Dynamic State Estimation



QP problem

$$\min_{\mathbf{x}} \frac{1}{2} \mathbf{x}^T G \mathbf{x} + g^T \mathbf{x}$$

$$\text{subject to } C_e \mathbf{x} + c_e = 0$$

$$C_i \mathbf{x} + c_i \geq 0$$

Cost function

$$\|A\mathbf{x} - \mathbf{b}\|^2$$

Opt. variables

$$\mathbf{x} = [\dot{\mathbf{q}}^{k+1} \quad \mathbf{w}_k \quad \boldsymbol{\tau}_k \quad \mathbf{f}_k]^T$$

Model error

$$\mathbf{w} = M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) - S\boldsymbol{\tau} - J^T(\mathbf{q})\mathbf{f}$$

$$A = [0 \quad I \quad 0 \quad 0] \quad \mathbf{b} = \mathbf{0}$$

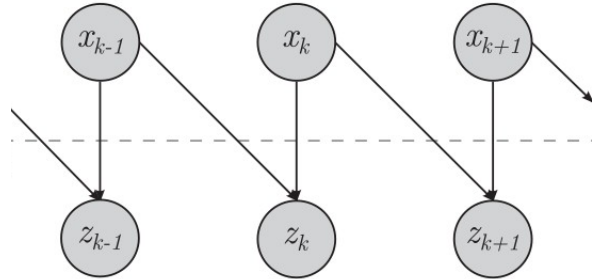
Measurement error:

$$A = \text{selector} \quad \mathbf{b} = \text{measures}$$

Dynamic state estimation using quadratic programming
X. Xinjilefu, S. Feng, and C. Atkeson – IROS 2014

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Two State Implicit Filter



TSIF

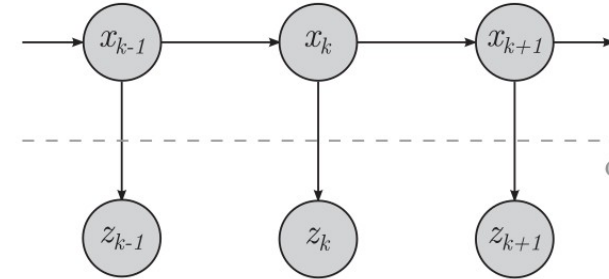
$$\| \mathbf{x}_k^{k-1} - \mathbf{x}_{k-1}^{k-1} \|_{P_{k-1}}^2 + \| \mathbf{b}_k + A\mathbf{x}_{k-1}^k + B\mathbf{x}_k^k \|^2$$

- Current and previous state (two-state) from measurement
- Model can be integrated as “measurement”
- Estimate as residual minimization

The Two-State Implicit Filter Recursive Estimation for Mobile Robots

M. Bloesch, M. Burri, H. Sommer, R. Siegwart, M. Hutter

Robotics and Automation Letters 2018



EKF

$$\text{p.s.} + \| F\mathbf{x}_{k-1}^k - \mathbf{x}_k^k \|^2 + \| H\mathbf{x}_k^k - \mathbf{z}_k \|^2$$

- Prediction from model
- Update from measurement
- Estimate as “weighted” mean between prediction and update

Any questions?

`mcamurri@robots.ox.ac.uk`